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Modeling of ship structural systems by events

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Motivation

Inclusion of the concept of ***entropy*** from information theory as **the only rational measure of** system **uncertainty,** for the estimation of structural properties.

Robustness and ***redundancy*** of systems of events may be considered as **additional characteristics** of engineering systems (ship structures).

The problem of ***distinction of complex engineering systems*** that have the **same reliabilities and probabilities of failure**, but different number of possible events.

Event oriented system analysis

System of events

$$\mathscr{S} = \begin{pmatrix} E_1 & E_2 & \dots & E_n \\ p(E_1) & p(E_2) & \dots & p(E_n) \end{pmatrix}$$

Events E_i , $i = 1,2,\dots,n$ are STATES of the system $\mathscr{S}$.

Entropy

Uncertainty of individual random event, *E*, with probability of occurrence $p(E)$ is measured by entropy:

$$H = -\log p(E)$$

Entropy of a system of events (Shannon):

$$H(\mathscr{S}) = H_n(\mathscr{S}) = H_n(p_1, p_2, \dots, p_n) = -\sum_{i=1}^{n} p_i \log p_i$$

Types of events of particular interest for engineering purposes:

- *Operational, E^o*
- *Failure, E^f*
- *Transitive, E^t*
- *Collapse, E^c*

System of events, $\mathscr{S}$, can be subdivided into subsystems: operational and failure

$$\mathscr{O} = \begin{pmatrix} E^o_1 & E^o_2 & \dots & E^o_N \\ p\left(E^o_1\right) & p\left(E^o_2\right) & \dots & p\left(E^o_{N_o}\right) \end{pmatrix} \quad \mathscr{F} = \begin{pmatrix} E^f_{N_o+1} & E^f_{N_o+2} & \dots & E^f_{N_o+N_f} \\ p\left(E^f_{N_o+1}\right) & p\left(E^f_{N_o+2}\right) & \dots & p\left(E^f_{N_o+N_f}\right) \end{pmatrix}$$

Then system, $\mathscr{S}$, can be viewed conditionally:

$$\mathscr{S}/\mathscr{O} = \begin{pmatrix} E_1^o/\mathscr{O} & E_2^o/\mathscr{O} & E_3^o/\mathscr{O} & \dots & E_N^o/\mathscr{O} \\ \frac{p\left(E_1^o\right)}{p\left(\mathscr{O}\right)} & \frac{p\left(E_2^o\right)}{p\left(\mathscr{O}\right)} & \frac{p\left(E_3^o\right)}{p\left(\mathscr{O}\right)} & \dots & \frac{p\left(E_{N_o}^o\right)}{p\left(\mathscr{O}\right)} \end{pmatrix}$$

$$\mathscr{S}/\mathscr{F} = \begin{pmatrix} E_{N_o+1}^f/\mathscr{F} & E_{N_o+2}^f/\mathscr{F} & E_{N_o+3}^f/\mathscr{F} & \dots & E_{N_o+N_f}^f/\mathscr{F} \\ \frac{p\left(E_{N_o+1}^f\right)}{p(\mathscr{F})} & \frac{p\left(E_{N_o+2}^f\right)}{p(\mathscr{F})} & \frac{p\left(E_{N_o+3}^f\right)}{p(\mathscr{F})} & \dots & \frac{p\left(E_{N_o+N_f}^f\right)}{p(\mathscr{F})} \end{pmatrix}$$

Entropy of a subsystem is:

$$H_{m_i}\left(\mathscr{S}/\mathscr{S}_i\right) = -\sum_{j=1}^{m_i} \frac{p\left(E_{ij}\right)}{p\left(\mathscr{S}_i\right)} \log \frac{p\left(E_{ij}\right)}{p\left(\mathscr{S}_i\right)}$$

- DOES NOT DEPEND on the probability of the system $p(\mathscr{S})$
- DOES NOT DEPEND whether the system is complete or incomplete

Robustness

Robustness is regarded as the system's capability to respond to all possible random failures uniformly.

The system robustness is related only to the failure modes of the system:

$$H_{N_f}\left(\mathcal{S}/\mathcal{F}\right) = -\sum_{i=N_o+1}^{N_o+N_f} \frac{p\left(E_i^f\right)}{p\left(\mathcal{F}\right)} \log \frac{p\left(E_i^f\right)}{p\left(\mathcal{F}\right)}$$

$$ROB\left(\mathcal{S}/\mathcal{F}\right) = ROB\left(\mathcal{S}\right) = H_{N_f}\left(\mathcal{S}/\mathcal{F}\right)$$

- $ROB\ (\mathcal{S}) = 0$, if there is no failure, or one failure event has a probability p=1
- $ROB\ (\mathcal{S})$ = max when all the failure modes are equally probable.
- Robustness increases with increasing number of events in the system

Redundancy

capability of a system to continue operations by performing different random operational modes of given probabilities in case of random failures of components

The system redundancy is related only to the operational modes of the system:

$$H_{N_o}(\mathscr{S}/\mathscr{O}) = -\sum_{i=1}^{N_o} \frac{p(E_i^o)}{p(\mathscr{O})} \log \frac{p(E_i^o)}{p(\mathscr{O})}$$

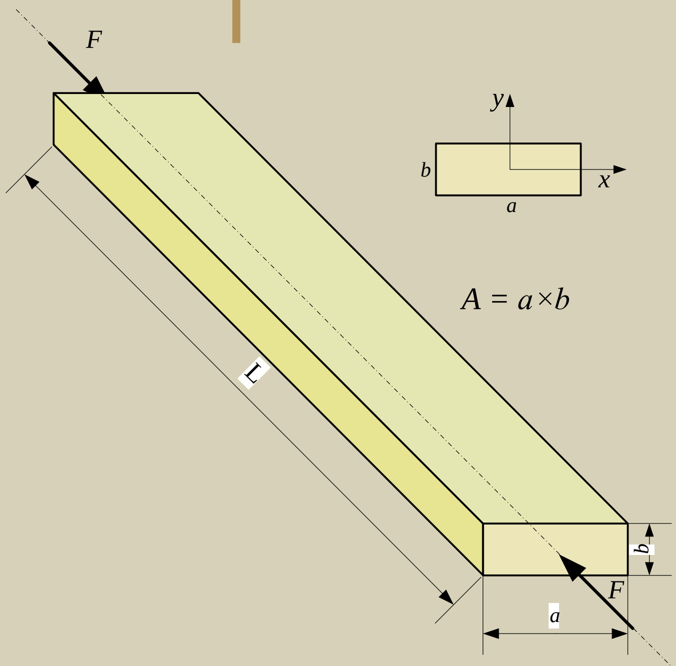
$$RED(\mathscr{S}/\mathscr{O}) = RED(\mathscr{S}) = H_{N_o}(\mathscr{S}/\mathscr{O})$$

RED = 0 if there are no operational states or if only one operational state is possible

RED = max. if all operational states are equally probable

Example 1

Robustness of a beam



		Mean	COV	Distribution
Side	a	36 mm	0,01	Normal
Side	b	25 mm	0,01	Normal
Length	L	500 mm	0,01	Normal
Modulus of elasticity	E	206000 N/mm²	0,01	Normal
Yield stress	σ_F	235 N/mm²	0,06	Log- Normal
Load (Force)	F_t	150 kN	0,3	Normal
Cross-section Area	A	900 mm²	0,1	Normal
Critical buckling stress (x)	σ_{Cx}	202,4 N/mm²	0,07	Log-normal
Critical buckling stress (y)	σ_{Cy}	219,3 N/mm²	0,07	Log-normal

Limit state functions:

Compression: $g_1 = A \cdot \sigma_F - F_t$

Buckling (x): $g_2 = A \cdot \sigma_{Cx} - F_t$

Buckling (y): $g_3 = A \cdot \sigma_{Cy} - F_t$

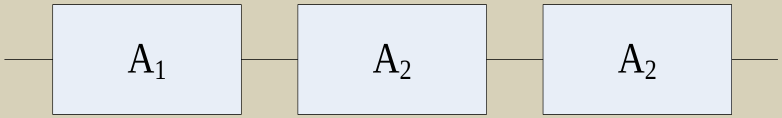
$\beta_{A1} = 3{,}125113$

$\beta_{A2} = 2{,}140569$

$\beta_{A3} = 3{,}152947$

Example 1

Series system, 3 basic events:



Compound events $E_i = 2^n = 8$

System of events:

$$\mathscr{S} = \begin{pmatrix} E_1^o & E_{1,1}^f & E_{2,2}^f & E_{3,3}^f & E_{1,2}^f & E_{1,3}^f & E_{2,3}^f \\ 0{,}982972 & 8{,}74 \cdot 10^{-4} & 1{,}53 \cdot 10^{-2} & 8{,}08 \cdot 10^{-4} & 1{,}44 \cdot 10^{-5} & 7{,}18 \cdot 10^{-7} & 8{,}08 \cdot 10^{-4} \end{pmatrix}$$

Reliability of series system:

$$R(\mathscr{S}) = p\left(E_1^o\right) = 0{,}\ 982972 \qquad p_f\left(\mathscr{S}\right) = 0{,}017028$$

Unconditional entropy: $H(\mathscr{S})$ = 0,134191 (2,807355; 0,047800)

System robustness: $ROB\left(\mathscr{S}\right) = H_{N_f}\left(\mathscr{S} / \mathscr{F}\right) = -\sum_{i=2}^{7} \frac{p\left(E_i^f\right)}{p\left(\mathscr{F}\right)} \log \frac{p\left(E_i^f\right)}{p\left(\mathscr{F}\right)} = 0{,}574081$

Maksimum robustness: $ROB_{\max}(\mathscr{S}) = \log(N_f) = 2{,}584963$

Event oriented system analysis of a beam

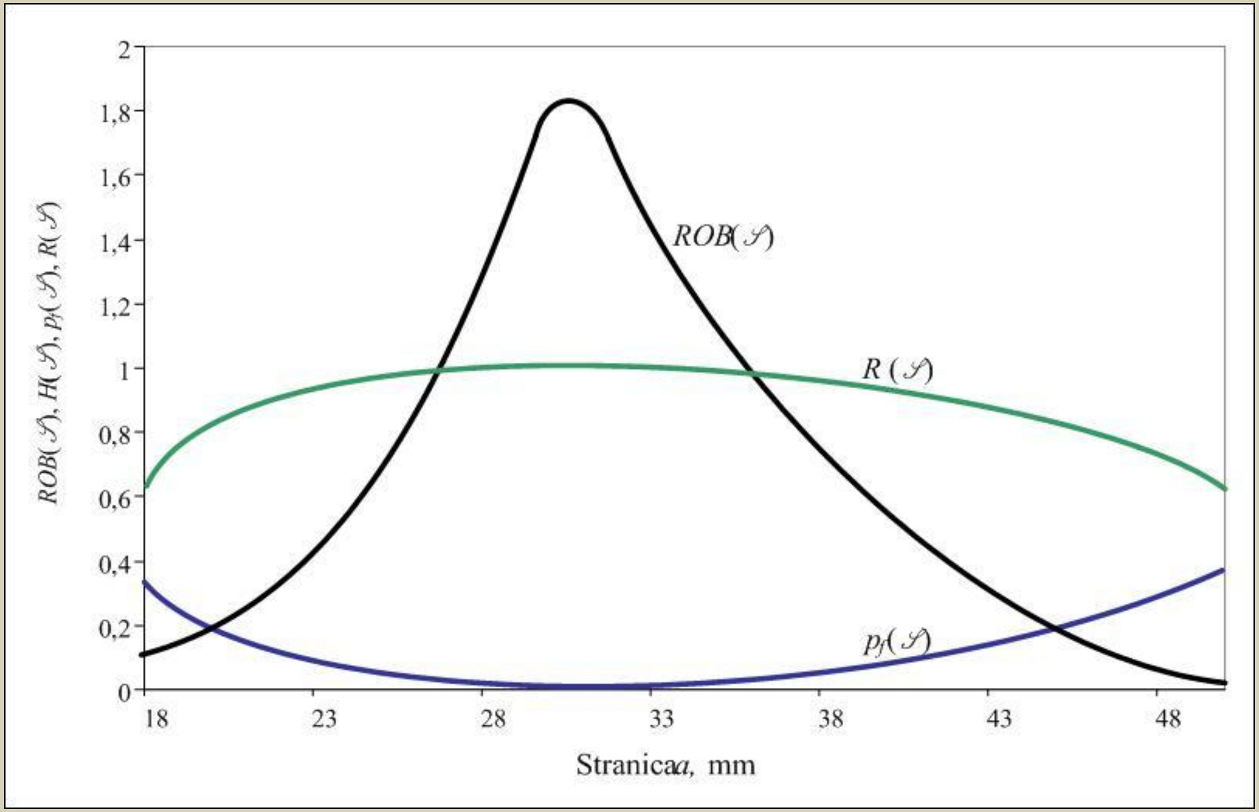
Requirements:

- $A = 900mm^2 = const.$
- $L = 500$ mm = const.

Results:

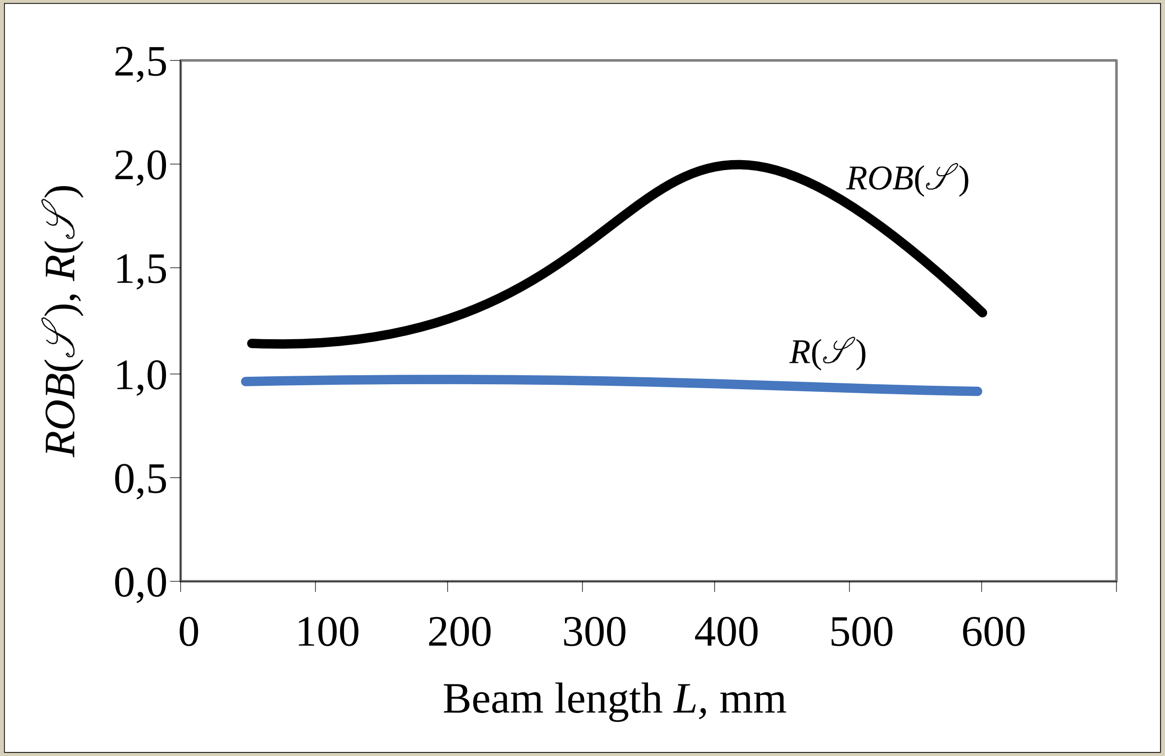
Max. *ROB* for
$a = b = 30$ mm

$ROB_{max} = 1{,}78033$



Example 1

For *a*=*b*=30 mm consider the change of L to robustness.



Condition:

$R\ (\mathscr{S}) > 0{,}999$

Result:

ROB_{max} for L = 423 mm

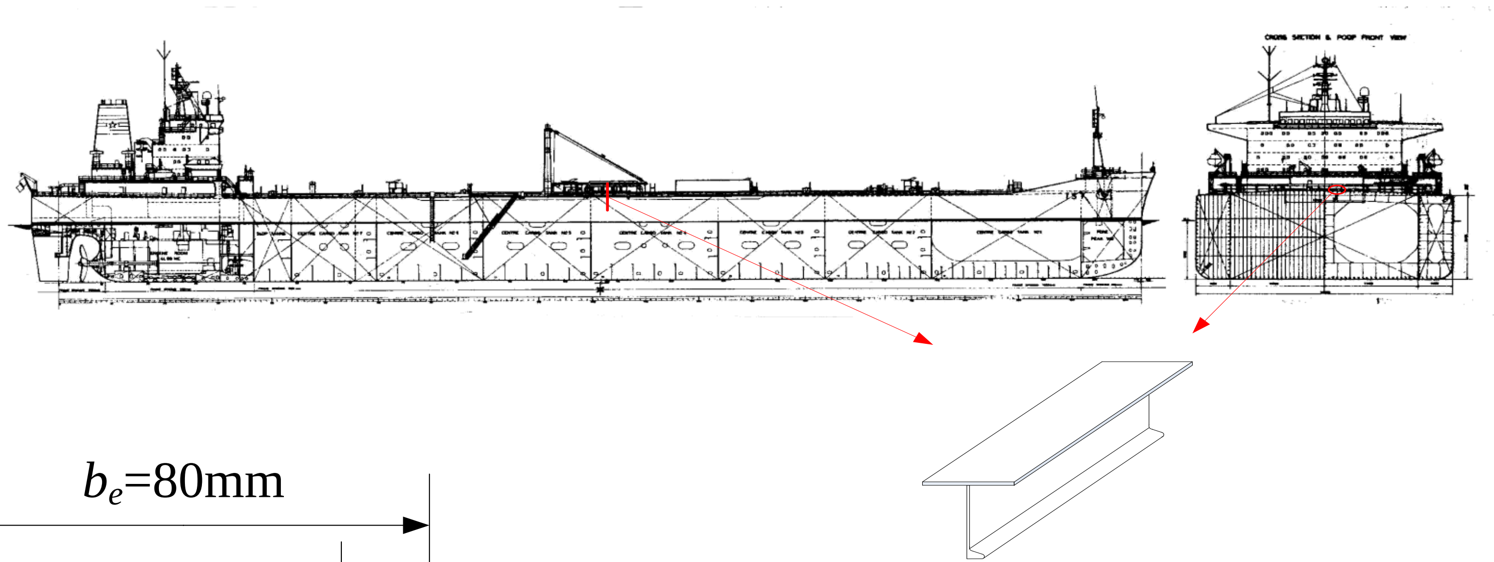
Conclusions for example 1

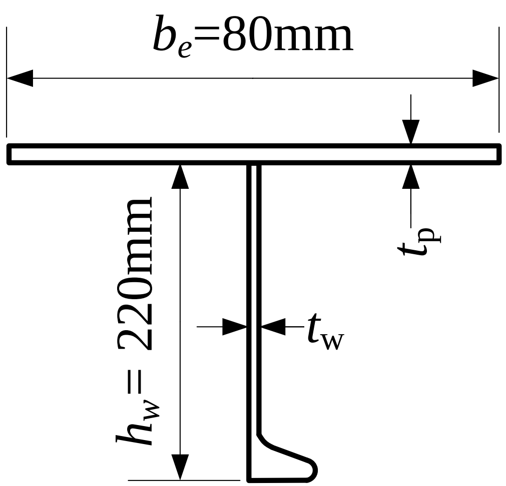
- Robustness changes for different dimensions.
- It is possible to calculate max. Robustness
- **For the same reliability level the robustness varies significantly.**

Example 2

EOSA: Robustness of ship structural elements

L = 173,15 m B = 31,4 m D = 11 m Δ = 47400 tdw





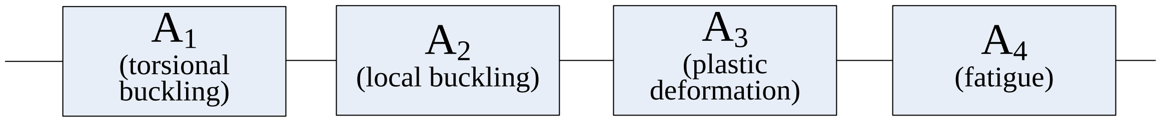
Loads: DnV

4 failure modes

- Torsional buckling
- Local buckling
- Plastic failure
- Fatigue failure

Example 2

Series system (4 basic events)



System:

$$\mathscr{S} = \begin{pmatrix} E_1^o & E_{1,1}^f & E_{2,2}^f & E_{3,3}^f & E_{4,4}^f & E_{1,2}^f \\ 0{,}90987 & 8{,}95\times10^{-2} & 2{,}90\times10^{-4} & 4{,}06\times10^{-4} & 5{,}82\times10^{-8} & 2{,}90\times10^{-4} \\ E_{1,3}^f & E_{1,4}^f & E_{2,3}^f & E_{2,4}^f & E_{3,4}^f & \\ 3{,}90\times10^{-4} & 5{,}77\times10^{-9} & 9{,}20\times10^{-5} & 1{,}85\times10^{-11} & 2{,}60\times10^{-11} & \end{pmatrix}$$

Reliability: $R\ (\mathscr{S}) = 0{,}909$ $\quad p_f(\mathscr{S}) = 0{,}090124$

Unconditional entropy: $H(\mathscr{S}) = 0{,}444712$

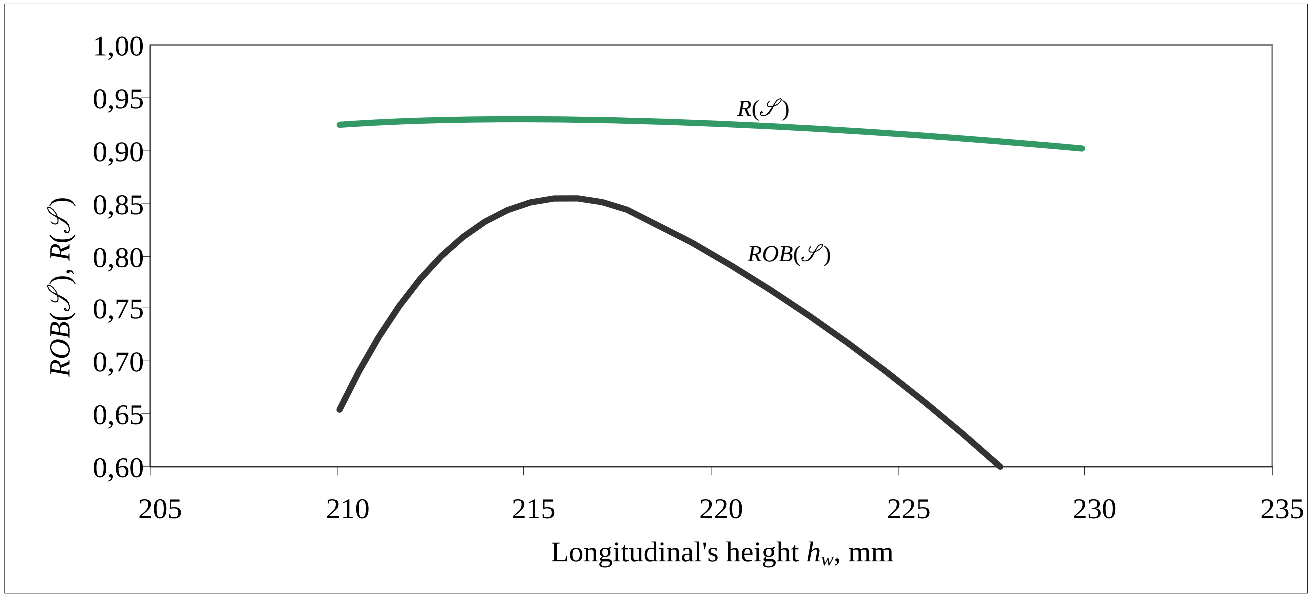
Robustness: $ROB\ (\mathscr{S}) = 0{,}804$

Max. Robustness = 3,321

Example 2

Analysis: Robustness change for different longitudinal's height and under conditions:

A = const. 210 mm < h_w < 230 mm R > 0,909



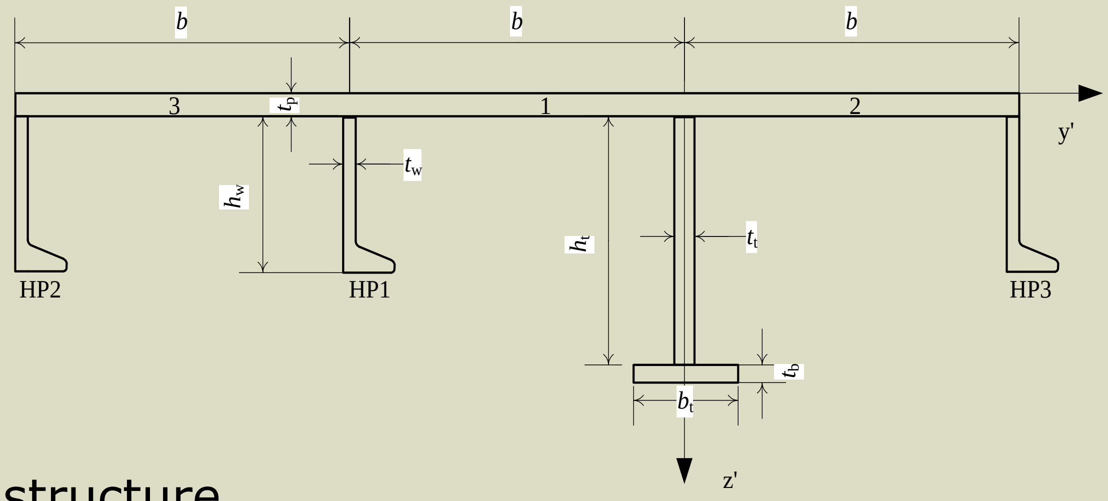
Conclusions:

- Robustness of a system of events can be related to design variables of ship structural elements
- max. *ROB* can be achieved within acceptable geometry limits
- it is possible to use ROB to compare different design solutions with various constraints applied

Example 3

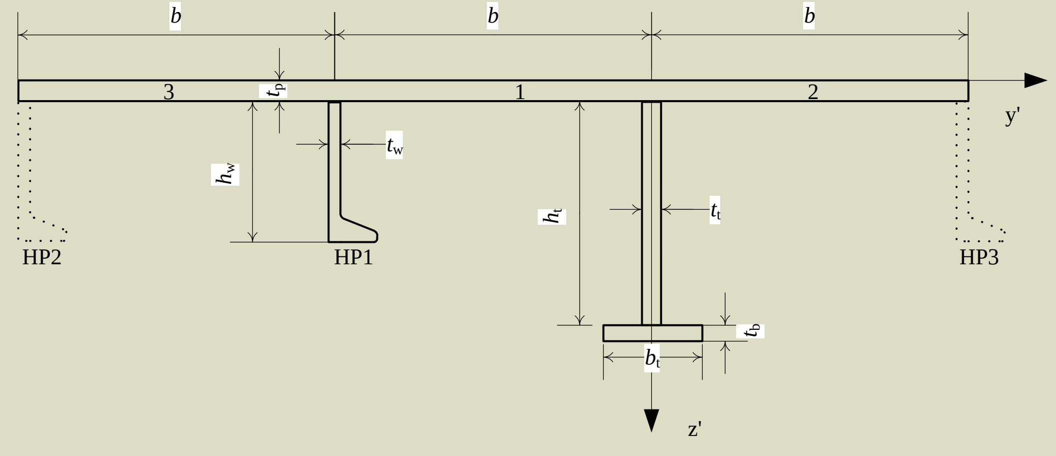
Redundancy of ship structures

Deck panel (tanker)



Functional levels:

1. Level – intact structure
2. Level – failure of of one longitudinal: no.2 or no.3
3. Level – failure of two longitudinals (non-redundant structure)



Example 3

Loads: DnV

Failure modes:

- buckling of plating between stiffeners (3)
- plastic failure of a deck girder (1)
- plastic failure of longitudinal (3)
- torsional buckling of a deck girder (1)
- torsional buckling of longitudinal (3)

1st operational level:

$$ {}^{1}\mathscr{S} = {}_{1}^{1}\mathscr{S} = \left({}_{1}^{1}\mathscr{S}^{i} + {}_{1}^{1}\mathscr{S}^{t} + {}_{1}^{1}\mathscr{S}^{c} \right) $$

n = 11 basic events, ${}^{1}N = 2^{11} = 2048$ compound events

2nd operational level:

$$ {}^{2}\mathscr{S} = \left({}^{1}\mathscr{S}^{i}, {}_{1}^{2}\mathscr{S} \bigcap {}^{1}E_{1}^{t}, \ldots, {}_{j}^{2}\mathscr{S} \bigcap {}^{1}E_{j}^{t}, \ldots, {}_{{}^{1}N^{t}}^{2}\mathscr{S} \bigcap {}^{1}E_{{}^{1}N^{t}}^{t}, {}^{1}\mathscr{S}^{c} \right) $$

15 operational states, ${}^{2}N$ = 9713 compound events

3rd operational level 18 operational states, ${}^{3}N$ = 12017

$$ {}^{3}\mathscr{S} = \begin{pmatrix} {}^{1}\mathscr{S}^{i} + {}^{1}\mathscr{S}^{c} + {}_{1}^{2}\mathscr{S} \bigcap {}^{1}E_{1}^{t} + {}_{2}^{2}\mathscr{S} \bigcap {}^{1}E_{2}^{t} + \ldots + {}_{15}^{2}\mathscr{S} \bigcap {}^{1}E_{15}^{t} + \\ + \left({}_{1}^{3}\mathscr{S} \bigcap {}^{2}E_{1}^{t} \right) \bigcap {}^{1}E_{1}^{t} + \left({}_{2}^{3}\mathscr{S} \bigcap {}^{2}E_{2}^{t} \right) \bigcap {}^{1}E_{1}^{t} + \left({}_{3}^{3}\mathscr{S} \bigcap {}^{2}E_{3}^{t} \right) \bigcap {}^{1}E_{1}^{t} + \\ + \left({}_{4}^{3}\mathscr{S} \bigcap {}^{2}E_{4}^{t} \right) \bigcap {}^{1}E_{2}^{t} + \left({}_{5}^{3}\mathscr{S} \bigcap {}^{2}E_{5}^{t} \right) \bigcap {}^{1}E_{2}^{t} + \left({}_{6}^{3}\mathscr{S} \bigcap {}^{2}E_{6}^{t} \right) \bigcap {}^{1}E_{2}^{t} + \\ + \ldots + \\ + \left({}_{18}^{3}\mathscr{S} \bigcap {}^{2}E_{18}^{t} \right) \bigcap {}^{1}E_{9}^{t} + \left({}_{18}^{3}\mathscr{S} \bigcap {}^{2}E_{18}^{t} \right) \bigcap {}^{1}E_{9}^{t} + \left({}_{18}^{3}\mathscr{S} \bigcap {}^{2}E_{18}^{t} \right) \bigcap {}^{1}E_{9}^{t} \end{pmatrix} $$

Operational levels, states and transitive events of a system

Example 3

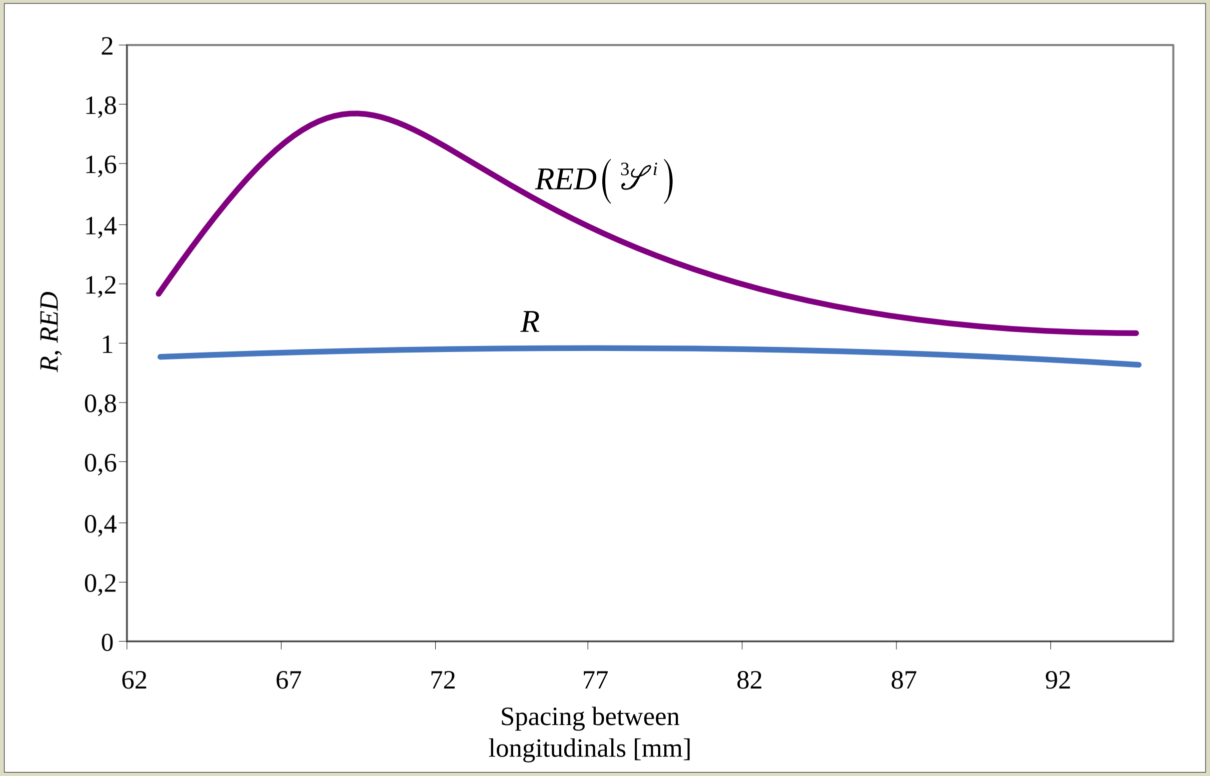
Analysis: Changes of the redundancy for different spacing between longitudinals.

Constraints:

- Reliability must be equal or larger than that of the original structural configuration.
- Weight of the panel = const.

Result:

$RED_{\max} = 1{,}776934$



Conclusions

Traditional probabilistic engineering system analysis based on physical and/or technical components of a system, may be extended by an EOSA.

Numerical examples confirm the relevance of EOSA and indicate potential improvements in system design (ROB, RED).

Problems

Possible numerical problems when dealing with larger systems .

For a complete event oriented system analysis an enumeration of ALL the possible events is needed.

General remarks

1. General relations among the probability, uncertainty of the system and uncertainties of the subsystems are derived by using information theory .
2. The uncertainties in system’s operation originate from the unpredictability of possible events.
3. The system uncertainty analysis is based on entropy.
4. The entropy, as the only rational measure of system uncertainty, does not depend on anything else other than possible events and in this sense is entirely objective.
5. The entropy of a system itself in general is not particularly helpful in the assessment of system performance.
6. The uncertainties of important subsystems of events, such as the operational and failure modes, as well as their relations to the uncertainty and reliability of the entire system, can provide a better insight into the system performance.
7. May be helpful in different fields of engineering in the refinement of system performance.